INTRODUCING WAVELETS IN TRAVELTIME TOMOGRAPHY: APPLICATION TO FOOTHILLS

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Summary

First-arrival traveltime tomography (FATT) applied to wide aperture seismic data has proved to be an interesting tool to investigate the earth structures. The resolution of FATT is controlled both by the theoretical resolution imposed by the first-arrival traveltimes, and the experimental device as well as the structure itself, which may lead to uneven ray coverage. The theoretical resolution power of FATT is limited by the size of the first Fresnel zone for each ray. In the area of limited ray coverage, there may be inconsistencies between the limited resolution power of FATT and the numerical sensitivity kernel of FATT corresponding to rays. Indeed, one raypath induces high frequency information along the ray, even at locations where resolution of FATT is expected to be really low. This can lead to models that have a high degree of data fitting, but which are far from reality or that contain artefacts. A classical way to account for the limited resolution of FATT is to augment the tomographic system with smoothing constraints. However, this kind of smoothing is difficult to tune and generally lacks adaptivity. Frequently, when the smoothing is tuned to remove the footprints of raypaths in poorly illuminated areas, short-scale features that could be resolved according to the resolution of FATT are smoothed accordingly, leading to a loss of information in the well-resolved areas.

The resolution of FATT generally varies locally. For example, considering surface acquisition geometries, the shallow structures will be sampled both by short- and long-offset rays running across the heterogeneities with different angles while the deeper structures will be sampled by long-offset rays only, hence a much narrower range of angles. This makes the resolution of FATT decrease fast with respect to depth. Two other factors creating non uniform ray coverage are the presence of low velocity anomalies which create shadow zones in the ray coverage and non uniform source and/or receiver arrays especially in the case of passive tomography.

To account for this varying resolution power, adaptive parametrizations based on non structured grids (Bohm et al. 2000) or multigrid (Zhou 2001) have been developed. The size of the elementary cells are adapted to the local ray coverage such that the amount of control to each model parameter tends to be uniform. Here, we proposed an alternative adaptive parameterization based on the wavelet transform (Mallat 1999). We discussed in an EAGE abstract (Delost et al. 2006) the possibilities offered by wavelet parametrization with a canonical synthetic example. We here present a comparison of synthetic example. We here present a comparison of the possibilities offered by wavelet parametrization with a canonical synthetic example. We here present a comparison of synthetic example. We here present a comparison of synthetic example. We here present a comparison of synthetic example. We here present a comparison of synthetic example. We here present a comparison of the possibilities offered by wavelet parametrization with a canonical synthetic example. We here present a comparison of nonsense.

Wavelet-based adaptive traveltime tomography

The slowness model is represented on a wavelet basis and the tomographic system is solved for the wavelet coefficients. The wavelet transform of the model provides a compact multigrid representation of the model thanks to orthogonal or bi-orthogonal basis functions built by translating (localization property) and dilating (scaling property) a mother wavelet and a scaling function. Generally, the resolution of each grid decreases by a factor 2 and the mesh spacing is set accordingly. The starting point is the slowness model on the finest grid of the multiresolution representation. The wavelet transform applies a cascade of orthogonal restrictions of the slowness model over different approximation grids. The restrictions of the slowness model on a coarse grid is encoded by the so-called scaling coefficients. At each step, the increment of information lost during the restriction on the coarser grid is encoded by the so-called wavelet coefficients before proceeding to the next grid. At the last iteration of the orthogonal transform, we end up with the wavelet coefficients on each grid plus the scaling coefficients on the coarser grid. The inverse wavelet transform proceeds in the other direction from the coarse grids to the finer ones. Multidimensional wavelet basis can be built by tensor product of 1D basis. The decomposition of the 2D slowness model of a wavelet basis can be written as

\[
\begin{align*}
\psi_{i,j}(x,z) &= \sum_{n=\infty}^{J} \sum_{m=\infty}^{I} \sum_{i=1}^{+\infty} \sum_{m=1}^{-\infty} c_{j,n,i,m}^{ww} \psi_{j,n}(x) \psi_{i,m}(z) \\
&+ \sum_{n=\infty}^{J} \sum_{m=\infty}^{I} \sum_{i=1}^{+\infty} \sum_{m=1}^{-\infty} c_{j,n,i,m}^{ww} \psi_{j,n}(x) \phi_{i,m}(z) \\
&+ \sum_{n=\infty}^{J} \sum_{m=\infty}^{I} \sum_{i=1}^{+\infty} \sum_{m=1}^{-\infty} c_{j,n,i,m}^{ww} \phi_{j,n}(x) \psi_{i,m}(z) \\
&+ \sum_{n=\infty}^{J} \sum_{m=\infty}^{I} \sum_{i=1}^{+\infty} \sum_{m=1}^{-\infty} c_{j,n,i,m}^{ww} \phi_{j,n}(x) \phi_{i,m}(z) \\
\end{align*}
\]

where \( \psi \) and \( \phi \) are the wavelet and scaling functions respectively. \( c_{j,n,i,m}^{ww}, c_{j,n,i,m}^{ww}, c_{j,n,i,m}^{ww}, \) and \( c_{j,n,i,m}^{ww} \) are the wavelet and scaling coefficients. The indices \( i, j, n, m \) are the scale and localization indices respectively. \( J \) and \( I \) denotes the coarsest levels of the multiresolution approximation. In this study, we used second generation wavelets constructed by a lifting scheme. They are called second generation wavelets by Wim Sweldens (Sweldens 1997) because they are not necessarily translation and dilate.
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tion of one fixed function. The lifting scheme provides a new way to construct biorthogonal wavelets in which boundary conditions can be easily added, allowing applications to intervals and domains. Implementation via lifting schemes is also a bit faster than classical wavelet transform algorithms.

The classic tomographic system is given by the following equation a):

\[ \Delta t = A \Delta u, \]

where \( \Delta t \) is the traveltime residuals, \( A \) is the sensitivity matrix and \( \Delta u \) are slowness perturbations. The wavelet parameterization leads to the modified system (2) b), where \( \Delta c = W \Delta u \) denotes the transformed slowness model. The matrix \( AW^T \) can be built by computing the 2D wavelet transform of each row of the original matrix \( A \) which contains digitized rays. Once system equation (2) has been solved, the perturbation model in the space domain is obtained by inverse wavelet transform of \( \Delta c \). The restriction of rays on a coarse grid is equivalent to extend its area of sensitivity (fig 1). The adaptivity on the parameterization can be simply implemented by applying a mask to each row of the sensitivity matrix \( AW^T \) by zeroing the fine-scale coefficients in the areas of the model where a poor resolution is expected. For surface acquisition, one may define an empirical rule relating the depth of a model parameter to the minimum source-receiver offset of the rays sampling this model parameter. The source-receiver offset provides an estimate of the width of the first Fresnel zone and, hence, of the theoretical resolution of the tomography. According to the sampling theorem, only the grids whose mesh spacing is two time smaller than the width of the first Fresnel zone will be locally involved in the inversion. Note that the wavelet transform has also the capacity to automatically adapt the parameterization to the resolution length of the gaussian functions to size of the first Fresnel zone associated with each ray, accordingly to the theory, using the equation:

\[ \tau = \sqrt{\lambda(z) \cdot offset(z)}. \]

\( \lambda(z) \) is the depth dependent wavelength expected in a one layer model and \( offset(z) \) is obtained using the ray cartesian equation. This method provides us a certain range of results depending on the weight we give to the regularization. For a low regularization setting, our optimal model (see fig 3 B) fits very well the first arrivals with the difficulties associated with FATT.

If no regularization is applied, the FATT results in a model that is very dependent on ray coverage (fig 3 A). Those raypaths are artefacts, i.e. they are not related to geological structures, hence the necessity of using regularization. Among classical methods, we choose smoothing using gaussian functions and we implemented a depth adaptive regularization : we related the correlation length of the gaussian functions to size of the first Fresnel zone associated with each ray, accordingly to the theory, using the equation:

\[ \tau = \sqrt{\lambda(z) \cdot offset(z)}. \]

Results with wavelets parametrization

Wavelets parametrization offers a large panel of basis functions featuring different properties. As the geometry of basis functions plays a relevant role in the reconstruction, geometry of the model is less dependent on raypaths footprints : rays tend to become larger as we go to coarser grids (fig 1). We applied Haar biorthogonal and
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linear interpolation scheme, which feature respectively one and two vanishing moments for both the real and the dual wavelet. We present results obtained with linear interpolation. Four levels of wavelets were implied, down to a coarse grid of 400m.

The model obtained (fig 3 D) features less ray footprints than model 3 B but is still not as smooth as model 3 C. Nevertheless, figures 2 C and 5 C show that the data fitting is better than for the smooth model, especially for short offsets.

We tried to quantify the resolution obtained by reconstructing structures of given sizes using checkerboard tests for models 3 C and 3 D. We used structures of 1 km and 2 km (fig 4 A and D respectively). The figure 4 E shows that with a smoothing similar to fig 3 C, we are able to reconstruct bigger heterogeneities, but the results are not as satisfactory with smaller heterogeneities, whose reconstruction is too smooth and with underestimated amplitudes (fig 4 B). The results obtained using wavelets tend to be more homogeneous over the different scales (4 C and 4 F), with less smearing and amplitudes that are conserved across the scales. The reconstruction in subsurface is more precise and small heterogeneities are better reconstructed. Big structures nevertheless are reconstructed by wavelets with less spatial coherence. But this is previsible given the coarsest scale involved in the wavelet decomposition corresponds to a 400 m grid interval. This interval is still significantly finer than the one that would be consistent with the theoretical resolution expected at this depth.

In the multiresolution decomposition, the number of grid points is divided by two for each level which imposes some limitations in the number of resolution levels, due to the size of the model. To overcome this limitation we will consider an artificially enlarged model, which will allow to involve coarser grids in the deep part of the model, leading to a smoother reconstruction in depth.

Conclusion

This study shows the interest of using wavelet parametrization in the traveltime tomography. It allows to have a better control of the resolution power depending on the parameter location. The wavelet parameterisation is more adaptive than classical methods and can lead to better control and image reconstruction in the tomography process.

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References


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Fig. 1: Projection of one ray over the fourth level of resolution.

Fig. 2: Time residuals depending on the distance source/receiver. A) for the low regularization model. B) for the smooth model. C) for the wavelet model. Short offsets are found along the diagonal.
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Fig. 3: Models obtained by FATT. A) without smoothing or wavelets, RMS error=15ms. B) low smoothing model, RMS=15ms. C) smooth model RMS=22ms. D) model obtained with wavelets, RMS=17ms.

Fig. 4: Checkerboard tests. A) perturbation model, size 1km. B) model reconstructed with smoothing. C) model reconstructed with wavelet parametrization. D) perturbation model, size 2km. E) model reconstructed with smoothing. F) model reconstructed with wavelet parametrization.

Fig. 5: Times picked and times calculated for shot n311. A) with low regularization. B) with high regularisation. C) with wavelets.
EDITED REFERENCES
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