H035

Accuracy of qP Wave Modeling in Anisotropic Acoustic Media by a Finite-difference Frequency-domain Method

A. Ribodetti* (IRD), S. Operto (Geosciences Azur - CNRS) & J. Virieux (LGIT-UJF)

SUMMARY

We assess the kinematic and dynamic accuracies of a finite-difference frequency-domain method for qP wave modelling in transversally isotropic acoustic media with tilted symmetry axis. This method was developed as a tool for frequency-domain full-waveform inversion which requires accurate travelt ime and amplitude modelling. The modelling method is based on the parsimonious mixed-grid method which requires 5 grid points per wavelength in homogeneous media to mitigate numerical dispersion. We compare seismograms computed with the acoustic frequency-domain method with that provided by the complete solution of the transversally isotropic elastic wave equation. As expected we observed strong travelt ime and amplitude mismatches in the case of strongly anisotropic materials such as zinc crystals. For weak anisotropy, we obtain a reasonable agreement although slight delay of the acoustic wide-angle reflections was observed in the case of a two-layer medium. The footprint of these inaccuracies in full-waveform inversion will need to be assessed before considering application to real data.
Introduction
It is now well established that general kind of seismic anisotropy resulting from material tilt of arbitrary orientation needs to be taken into account in seismic imaging for accurate reservoir characterisation in complex environment. We recently presented a finite-difference frequency-domain (FDFD) method for qP wave modelling in 2D acoustic transversely isotropic media with tilted symmetry axis (TTI medium) (Operto et al., 2007). We developed this modelling scheme as a tool for frequency-domain full-waveform inversion of wide-aperture seismic data. So far, the accuracy of the numerical stencil was mainly assessed thanks to a classic harmonic dispersion analysis in homogeneous media which suggested that a discretisation rule of 5 grid points per wavelength should provide kinematically-accurate solutions in homogeneous media. In this study, we provide new insights on the relevance of the acoustic anisotropic FDFD method from both a kinematic and dynamic view points by comparison with the full solution of the 2D transversally isotropic (TTI) elastic wave equation. This later solution was computed with a staggered-grid finite-difference time-domain (FDTD) method. The FDFD method was developed for the anisotropic acoustic wave equation recently introduced by Zhou et al. (2006). In the following, we briefly review the TTI acoustic wave equation before presenting numerical examples for different anisotropic materials to assess the validity range of the FDFD method.

Method
The modified 2D TTI acoustic wave equation proposed by Zhou et al. (2006) is given by

\[
\frac{1}{\kappa_0} \frac{\partial^2 q}{\partial t^2} - (1 + 2\delta)\frac{\partial q}{\partial t} - H_0 \frac{\partial q}{\partial \theta} = (1 + 2\delta)Hq - 2(\epsilon - \delta)\frac{\partial^2 q}{\partial \theta^2} \tag{1}
\]

with

\[
H = \cos^2 \theta_0 \frac{\partial}{\partial x} \left( b \frac{\partial}{\partial x} + \frac{\partial}{\partial z} \right) + \sin^2 \theta_0 \frac{\partial}{\partial z} \left( b \frac{\partial}{\partial z} + \frac{\partial}{\partial x} \right) - \frac{\sin 2\theta_0}{2} \left( \frac{\partial}{\partial x} \frac{\partial}{\partial z} + \frac{\partial}{\partial z} \frac{\partial}{\partial x} \right)
\]

\[
H_0 = \sin^2 \theta_0 \frac{\partial}{\partial x} \left( b \frac{\partial}{\partial x} + \frac{\partial}{\partial z} \right) + \cos^2 \theta_0 \frac{\partial}{\partial z} \left( b \frac{\partial}{\partial z} + \frac{\partial}{\partial x} \right) + \frac{\sin 2\theta_0}{2} \left( \frac{\partial}{\partial x} \frac{\partial}{\partial z} + \frac{\partial}{\partial z} \frac{\partial}{\partial x} \right) \tag{2}
\]

where \( \theta_0 \) is the angle of the symmetry axis. \( \epsilon \) and \( \delta \) are Thomsen dimensionless anisotropic parameters (Thomsen, 1986). \( \kappa_0 \) is the bulk modulus on the symmetry axis. \( q \) is the pressure wavefield and \( p \) is an auxiliary wavefield introduced to reduce the original 4th-order differential equation of Alkhalifah (1998) to a system of 2nd-order differential equations (Zhou et al., 2006). The first equation governs the propagation of the wavefront (elliptical part) and the second one compensates for loss of anisotropy for TTI media not only in the lateral but also in the depth directions. The particular cases \( \theta_0 = 0^\circ \) and \( \theta_0 = 90^\circ \) degenerate into equations for VTI and HTI media respectively. The case \( \epsilon = \delta \) degenerates into a single second-order differential equation for elliptic media. The system 1 was discretized with the so-called parsimonious mixed-grid FDFD method. Details and references can be found in Operto et al. (2007).

Numerical examples
Simulation in homogeneous media
We present two simulations in VTI homogeneous media. The first medium represents a highly-anisotropic zinc crystal (Tab. 1). Such medium was considered for numerical modelling in Carcione (2007); Komatitsch et al. (2000) at the ultrasonic scale. In this study, we perform the simulation at the seismic scale to avoid unstabilities during LU factorization of system 1. The source is an explosion with a Ricker wavelet of central frequency 17 Hz. The medium was discretized with a 201 \( \times \) 201 FD grid with a mesh interval of 15 m which represents 4 grid points per wavelength at 50 Hz. The source is on the middle of the grid. A horizontal line of receivers (green lines in Fig. 1) is set 990 m above the source. Analytical qP and qS wavefront curves
Model | $c_{11}$ | $c_{13}$ | $c_{33}$ | $c_{55}$ | $c_0$ (km/s) | $\delta$ | $\epsilon$ | $\theta(\circ)$ | $\rho$ (kg/m$^3$) |
---|---|---|---|---|---|---|---|---|---|
zinc | 16.5 | 5 | 6.2 | 3.96 | 2.955 | 2.71 | 0.831 | 0 | 7100 |
sed | 4.8 | 1.413 | 4 | 1.333 | 4 | 0.02 | 0.1 | 0 | 2500 |
Layer 1 | 1.921 | -0.533 | 1.921 | 1.227 | 2.955 | 0.0 | 0 | 0 | 2200 |
Layer 2 | 6.972 | 3.136 | 5.81 | 1.394 | 4.821 | 0.02 | 0.1 | 0 | 2500 |

Table 1: Properties of the 2 homogeneous media (zinc and sed) and of the two layers L1 and L2 of the heterogeneous medium. The $c_{ij}$ coefficients are in $10^{10}$ Nm$^{-2}$.

(Carcione, 2007) are superimposed on the acoustic pressure wavefields at a time of 0.36 s in Fig. 1a. We observe a good kinematic agreement on the symmetry axis and on the perpendicular direction but some advance of the numerical wavefront in between these two directions. This is confirmed by the comparison between acoustic FDFD and elastic FDTD seismograms where both traveltimes and amplitude mismatches between the two sets of pressure seismograms are observed at large offsets (Fig. 1b). The accuracy of the FDFD solution on the symmetry axis is further confirmed by comparison with analytical seismograms whose expression is given in Carcione (2007) (Fig. 1c). This first test confirms the poor kinematic and dynamic accuracy of the VTI acoustic wave equation for highly-anisotropic materials.

We next consider an anisotropic material representative of sedimentary rocks ($\delta = 0.02$ and $\epsilon=0.1$) (referred as sed in Table 1). The dimension of the FD grid and the grid intervals are the same that of the previous experiment. The central frequency of the Ricker wavelet is 30 Hz. The FDFD solutions match well both the analytic wavefront curves (Fig. 1d) and the FDTD seismograms (Fig. 1e). We also present a simulation in the previous sedimentary material but with a $45^\circ$-tilted symmetry axis. The receiver line was rotated accordingly to be able to compare the TTI acoustic seismograms with the elastic ones previously computed in a VTI configuration (Fig. 2a). The TTI acoustic seismograms match equally well the elastic seismograms (Fig. 2b).

**Simulation in heterogeneous media**

We present a simulation in a two-layer medium whose properties are given in the 2 bottom lines of Table 1. The interface between the two layers and the symmetry axis of the anisotropy are vertical. The layer where the source is excited is isotropic. The explosive source whose wavelet is a Ricker of central frequency 17 Hz is located at position $(0,0)$ in Fig. 3(a-b). The dimension of the FD grid is 681 $\times$ 641 with a mesh interval of 10 m. Seismograms are computed for a vertical receiver line in the isotropic layer and for a horizontal receiver line crossing the interface 800 m below the source (green lines in Fig. 3(a-b)). The acoustic wavefield contains the direct, reflected, transmitted and head wave arrivals (Fig. 3(a-b)). A good agreement between acoustic and elastic seismograms is observed for the horizontal receiver line (Fig. 3(c-e)) while a slight delay of the wide-angle reflection is observed in the acoustic seismograms (Fig. 3(d-f)). This again illustrates some inaccuracies of the acoustic simulation for propagation directions oblique with respect to the symmetry axis.

**Conclusions**

We assessed the accuracy of an anisotropic acoustic FDFD method by comparison with full elastic solutions computed with a FDTD method. As expected, we observed strong mismatches between acoustic and elastic solutions in strongly-anisotropic media such as zinc crystals for oblique incidence angles with respect to the symmetry axis. For weakly-anisotropic media, we observed a good agreement between acoustic and elastic seismograms in homogeneous media. Simulation in a two-half-space medium reveals a reasonable accuracy of the acoustic solution although we note a slight delay in the acoustic wide-angle reflection. The footprint of these inaccuracies in full-waveform inversion of elastic synthetic data as well as that of artificial S waves excited by the anisotropic acoustic wave equation (Grechka et al., 2004) will need to be assessed before considering application to real data.
Figure 1: a) Wavefield in the zinc model. Velocity (red) and wavefront (blue) curves are superimposed. b) Comparison between elastic FDTD (solid) and acoustic (dash) FDFD seismograms. c) Comparison between acoustic FDFD and analytic seismograms on the symmetry axis for offsets 0.9 and 1.6 km. d) Wavefield in the weak-anisotropy model at 0.33 s. e) Same as (b) for the weak-anisotropy model.

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References
Figure 2: a) Same as Fig. 1d but for a tilt of 45°. b) Comparison between seismograms computed in TTI acoustic (dash lines) and VTI elastic (solid lines) media. The 2 sets of seismograms are not strictly at the same offsets because of the rotation of the receiver line in the TTI simulation.

Figure 3: a) Elastic wavefield. The isotropic layer is on the left. The interface is the thin black line. b) Acoustic wavefield computed with the FDFD method. (c-d) Elastic and acoustic seismograms for lines L1 (c) and L2 (d). (e-f) Comparison between elastic (solid) and acoustic (dash orange) seismograms for receiver lines L1 (e) and L2 (f). Residuals are the dash red lines.