Robust trajectory design using invariant manifolds. Application to (65803) Didymos

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The AIDA mission
Uncertainties and nonlinearities in space missions

Radar imaging of 101955 Bennu

Landing of Philae
Are there safe orbits in the Didymos system?

Failure of the propulsion system? Can we place a CubeSat?
Outline

1. Safe orbits in the Didymos system

2. Robust mission design using manifolds

3. Application to the Didymos system
1. Safe orbits in the Didymos system

2. Robust mission design using manifolds

3. Application to the Didymos system
1. High fidelity propagator of the Didymos system
1. Binary model: full three body problem

Orbital period \( \sim \) 1/2 day
Eccentricity: max 0.03

\[ \text{Didymain} \quad (790 \times 790 \times 761 \text{ m}) \]

\[ \text{Didymoon} \quad (103 \times 79 \times 66 \text{ m}) \]

Mass parameter \( \sim 1\% \)

1. Various stable orbits in the CRTBP are considered

**Triangular points**
- L4
- L5

**Terminator orbits**

**Planar periodic orbits**
- (a) Direct interior.
- (b) Direct exterior.
- (c) Retrograde interior.
- (d) Retrograde exterior.
- (e) CSO.
1. Robustness is assessed with a Monte Carlo analysis

- \( C_r \in [1, 2] \)
- \( \frac{m}{S} \in [20, 40] \frac{\text{kg}}{\text{m}^2} \)
- \( \phi(0) \in [-10, 10] \text{ deg} \)
- \( e(0) \in [0, 0.03] \)
- \( \theta(0) \in [0, 360] \text{ deg} \)

- 1000 samples
- Uniform distributions
- No correlation
1. Triangular points are not safe
1. Most planar orbits are not safe

Planar Periodic Orbits
- Close to the small bodies
- Subject to strong gravity field signals

But…
- Frequent eclipses
- No observation of the asteroid system

Similar results for: direct interior, direct/retrograde exterior, CSO
1. Retrograde interior orbits are safe!

Planar Periodic Orbits
- Close to the small bodies
- Subject to strong gravity field signals

But…
- Frequent eclipses
- No observation of the asteroid system
1. Retrograde interior orbits are safe!

- Planar Periodic Orbits
  - Close to the small bodies
  - Subject to strong gravity field signals

- But…
  - Frequent eclipses
  - No observation of the asteroid system

The diagram shows the failure events with different markers indicating different conditions. The graph plots $P_{\text{failure}}(x_0)$ versus $x_0$ [km], with additional axes for $C_{r,m}/B$ [m$^2$/km] and $\theta(0)$ [deg].
1. Terminator orbits are safe!

Terminator Orbits
- Near-circular orbits perpendicular to the Sun
- Remain perpendicular to the Sun due to SRP
- Allow observation of the binary system

View from the Sun

Side view
3D orbit

Impact Primary
Impact Secondary
Escape
No failure

\( P_{\text{failure}} (a_0) \)

2:1 resonance

\( a_0 \; [\text{km}] \)
1. Terminator orbits are safe!
1. Safe orbits in the Didymos system

2. Robust mission design using manifolds

3. Application to the Didymos system
2. Can we guarantee where the motion will evolve?
2. Invariant manifolds of an orbit

Quasi-periodic tori

Stable periodic orbit

Unstable manifold

Unstable periodic orbit

Numerical computation: solution of a PDE
2. Invariant manifolds as bounds for the motion
2. Uncertainties can change the manifold
2. Lyapunov-like stability to accommodate uncertainties

Take all possible realizations

The motion is bounded by this union

If initial conditions are in this intersection
2. Trade-off between large intersection and narrow union

\[
\min_p J(I(p), U(p)) \quad \text{s.t.}
\]

\[I(p) \supseteq I \quad \text{Injection accuracy requirements}\]

\[U(p) \subseteq U \quad \text{Collision avoidance}\]

Min bounds of the motion

Max tolerable uncertainty in initial conditions
Outline

1. Safe orbits in the Didymos system

2. Robust mission design using manifolds

3. Application to the Didymos system
3. Design of robust interior orbits in the Didymos system

20% uncertainty in mass parameter

No impact

1 cm/s uncertainty
3. Unperturbed solution: 2d torus

Technical need: compute manifold passing through a desired point
3. Safest orbit given uncertainty in the injection velocity

Objective function \( = U \)

Min distance from bodies

Worst-case trajectories

Nominal initial conditions
3. Maximum tolerable uncertainty in the injection velocity
3. Ballistic landing from $L_2$

20% uncertainty in mass parameter

We consider only the first impact
3. Libration point $L_2$ (unstable)
3. Manifolds of $L_2$: minimum energy landing
3. Lyapunov orbit (unstable)
3. Asymptotic trajectory of the Lyapunov orbit
3. Manifolds of the Lyapunov orbit

![Diagram showing manifolds of the Lyapunov orbit]

- **Stable manifold**
- **Unstable manifold**
- **Lyapunov orbit**
3. How to guarantee that a trajectory will land?

If the lander is deployed from this point with the same energy of the manifold, Feasible deployment directions.
3. High energy allows to accommodate more uncertainty
3. Feasible set of the velocity relative to the manifold

- Min energy
- Feasible set
- Max energy

Max y-crossing at L2

Envelope of transit orbits
3. Maximization of the uncertainty in the initial velocity

Max tolerable uncertainty in velocity

Guaranteed impact location

\[ v_x \text{ [cm / s]} \]

\[ v_y \text{ [cm / s]} \]

\[ x \text{ [m]} \]

\[ y \text{ [m]} \]
3. Monte Carlo validation of a deployment from $y = 0.5$

Zero-velocity curves for maximum energy
Conclusions

Two families of **safe orbits** exist: interior retrograde and terminator orbits

Invariant manifolds are used to **bound** the motion

Mission design: **trade-off** between large initial state uncertainty and narrow bounds

Way forward:

- accommodating **binary eccentricity** and **uncertainties polyhedron**
- **Nonlinear** Conley criterion for the landing
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